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On Abbe's Theory of Image Formation in the Microscope

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On Abbe's theory of image formation in the microscope†

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Abstract. A historical survey is given of publications from 1873 to 1910 concerning Abbe's theory of image formation in the microscope. Furthermore, the theory is presented in short by the algorithm of the complex Fourier transform.

1. Introduction

Abbe's 108 year old theory of image formation in the microscope undoubtedly contains essential elements of modern 'coherent optics' and holography. But anyone interested in optics who wants to reconstruct the relationships will encounter considerable difficulties, because so far a consistent representation of Abbe's theory in the modern mathematical style does not exist. It must be reconstructed from separate publications of the years 1873 to 1910.

A detailed representation of the mathematical derivation of Abbe's theory will be published in *Zeiss Information* [1]. This paper gives a historical survey of Abbe's theory from a modern point of view, and deals with some mostly unknown facts.

2. History

Abbe's first publication of 1873 [2], which has become famous, is merely a verbal treatment of the diffraction theory of microscopic imaging, the sine condition, the laws of energy transport in the microscope (constancy of luminance), and the principles of the desired correction of microscopic objectives, but without mathematical derivations.

He states with regard to the diffraction theory of microscopic imaging: "The limit of discrimination will never appreciably exceed a whole wavelength under central illumination ... and half a wavelength ... under extreme oblique illumination." This means

$$\Delta = \lambda/(n \sin \sigma) \text{ or } \Delta \geqslant \lambda/(2n \sin \sigma).$$

He also describes in detail his experiments with respect to the resolution of periodic structures. The most important results of these experiments are given in brief below.

If due to diaphragms introduced in the back focal plane of a microscope objective only one order of the diffraction spectrum of a periodic object is allowed to pass through, no object structure can be recognized in the 'secondary image', that is when looking through the eyepiece. A faithfully represented structure of an object can only be recognized when at least two adjacent orders of the diffraction spectrum are

† Excerpt from a lecture given at the Symposium of the International Commission for Optics (ICO) and the Deutsche Gesellschaft für angewandte Optik (DGaO) on 13 June, 1973 in Aalen, West Germany.

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allowed to pass through. If two non-adjacent orders of the diffraction spectrum pass through, the structure of the image is not faithful to the object. The period length of the image is, for example, twice that of the object.

In [2] Abbe announced that the mathematical derivation of these verbally presented results would be published in *Jenaische Zeitschsift für Naturwissenschaft*, but it never appeared. In 1874 Helmholtz [3] published an independent investigation which proved the sine condition. He used a different scientific approach to treat the influence of diffraction on microscopic imaging, and produced a mathematical derivation. In a paper of 1880 [4] Abbe rejected unjustified criticism of his theory by R. Altmann, and announced again the early publication of a mathematical proof of his theory.

In 1882, in close co-operation with Abbe, Dippel [5] published in *Handbuch der allgemeinen Mikroskopie* (handbook of general microscopy) a comprehensive account of Abbe's theory, but still without mathematical derivations and not going substantially beyond the contents of the original publication. This excellent description was readily accepted and widely disseminated. It was included in this form in almost all handbooks, and for a long time remained the most important source of Abbe's theory. During Abbe's lifetime only the mathematical proof of the sine condition was published, in an article in a handbook [6] by S. Czapski, then Abbe's closest collaborator and later his successor.

Abbe died in 1905 without having published the mathematical proof of his diffraction theory of image formation in the microscope. A first clue to the existence of such a proof can be found in an article by Lummer, published in Müller-Pouillet's handbook of physics in 1909 [7], 4 years after Abbe's death. In connection with the elementary discussion of Abbe's theory, Lummer writes that he heard about the mathematical derivation in 1888 from Abbe himself during a private lecture in Jena. He also mentions that on account of its complexity it is not suitable for publication in a handbook of this kind.

In 1910, with the permission of Mrs. Abbe, Lummer and Reiche presented Abbe's theory in a consistent mathematical formulation [8] according to Abbe's lecture scripts of the year 1888. Our knowledge of the mathematical formulation of Abbe's theory is exclusively based on this paper by Lummer and Reiche. According to this paper, Abbe correctly calculated the images of the following objects produced by the microscope:

(*a*) an infinitely narrow slit in coherent and incoherent light;

(*b*) two infinitely narrow slits at a finite distance in coherent light as a function of the angle of incidence of the illumination; and

(*c*) two infinitely narrow slits at a finite distance in incoherent light.

Abbe proved by these examples that in *coherent* light parallel to the axis the double-slit structure is still recognizable if the slit width is $\Delta \geqslant \lambda/n \sin\sigma$, whereas in incoherent light $\Delta \geqslant \lambda/2n \sin\sigma$. The same resolution can be obtained with coherent illumination when the angle of incidence of the illumination is such that the light disturbance† in the two slits has a phase difference of an odd multiple of $\frac{\pi}{2}$.

Abbe also discussed coherently illuminated slits of finite width. He is the first to use in optics the Fourier integral in the form presently known as the 'Fourier cosine transform' (see the reproduction of the corresponding page from the book by

† 'Light disturbance' is a scalar value of the electromagnetic field. It may be the complex amplitude of one component of the electric or magnetic field vector.

$$f(x) = \frac{1}{\pi} \int_{0}^{\infty} dz \int_{-\infty}^{+\infty} f(u) \cos z (u - x)\, du \quad . \quad . \quad . \quad . \quad (52)$$

Die gesuchte Funktion ist also:

$$f(x) = \frac{1}{\pi} \int_{0}^{\infty} dz \int_{-a}^{+a} \cos z (u - x)\, du$$

$$= \frac{1}{\pi} \int_{0}^{\infty} dz\, \frac{2}{z} \sin (a z) \cos (z x),$$

oder wenn man noch $a z = w$ **setzt:**

$$f(x) = \frac{2}{\pi} \int_{0}^{\infty} dw\, \frac{\sin w \cos \left(\frac{x}{a} w\right)}{w}$$

$$= \frac{1}{\pi} \int_{-\infty}^{+\infty} dw\, \frac{\sin w \cos \left(\frac{x}{a} w\right)}{w} = J_0.$$

Figure 1. Fourier integral as used by Abbe in his theory of the microscope (reproduced from [8]).

Lummer and Reiche [8] (figure 1)). It can be assumed that Abbe himself used the term Fourier integral in his own research work and in his lectures; it was not added later in the adaptations by Lummer and Reiche. This is particularly suggested by a hint Dippel gives in his handbook of general microscopy [5], where he says that an exact representation of the theory of image formation in the microscope would require the use of Fourier integrals, which in this context, however, would lead one astray. The chapter on image formation in the microscope in Dippel's handbook was prepared as early as 1882 in close co-operation with Abbe, that is 6 years before the lecture in Jena when Lummer heard from Abbe about the theory of image formation in the microscope. This little known fact proves that Abbe can be named with every justification as the founder of Fourier optics and thus also a pioneer in holography.

According to Lummer and Reiche, Abbe used the Fourier cosine transform also to prove that the sequence of integrations over object and aperture can be interchanged when calculating microscopic images, which explains the experiments described in [2].

Abbe himself did not mathematically formulate this step. At the suggestion of Lummer, Wolfke, one of Lummer's pupils, made the formulation in 1909 [9]†.

† He is the same Wolfke who in 1920 was the first to indicate in a theoretical paper [10] the potentialities of holography to reveal molecular structures in crystalline lattices. According to his ideas a 'hologram' should be taken in X-ray light and reconstructed in visible light. Although this was never realized, it demonstrates the close relationship between the theory of image formation in the microscope and the principles of holography.

Wolfke's derivation can be found in many handbooks of theoretical optics (see, for example, [11, 12]).

3. Abbe's theory in terms of the algorithm of the complex Fourier transform

Following the above historical survey, the propositions of Abbe's theory included in the book by Lummer and Reiche are derived below using the calculus of the complex Fourier transform. All calculations refer to the schematic microscope beam path shown in figure 2.

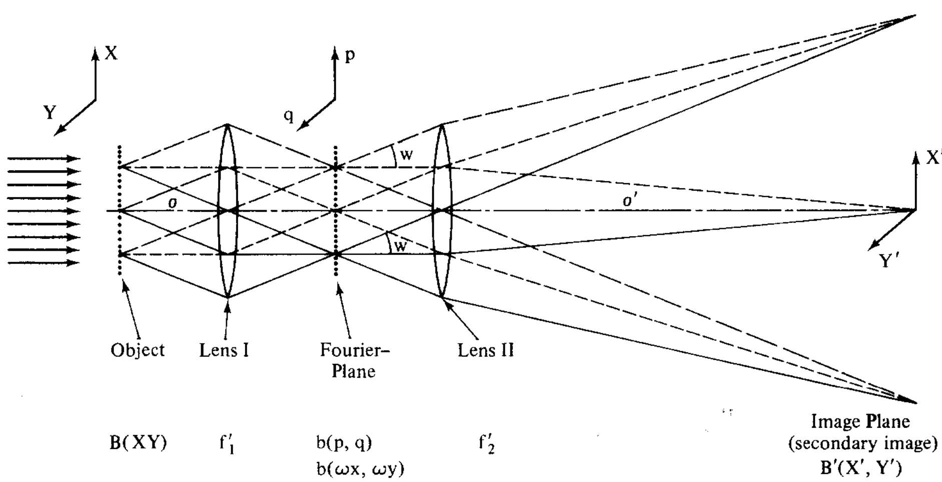

Figure 2. Light path for microscopic image formation in coherent light.

Let the object function $B(X, Y)$ be a light disturbance function† which results when a transparent object, whose transmittance function $A(X, Y)$ is different from zero only within the limits $X=\pm a$ and $Y=\pm b$, is illuminated by coherent parallel light which is in the general case incident at an angle with components α_x and α_y. Let the local limitation of the transmittance function be formulated by the Π-function‡. The object function will thus have the form

$$B(X, Y)=\Pi\left(\frac{X}{2a}, \frac{Y}{2b}\right)A(X, Y)\exp\left[-ik(X\sin\alpha_x+Y\sin\alpha_y)\right], \tag{1}$$

with

$$k=\frac{2\pi}{\lambda}. \tag{2}$$

† See footnote on page 1692.

‡ $\Pi(z)=\begin{cases}0, & \text{if } |z|>\frac{1}{2}\\ 1, & \text{if } |z|<\frac{1}{2}\end{cases}, \quad \Pi(x,y)=\Pi(x)\Pi(y).$

The distribution function of the (complex) light amplitude in the back focal plane (coordinates p, q) is the Fourier transform $b(\omega_x, \omega_y)$ of the object function $B(X, Y)$ with the Fourier frequencies $\omega_x = 2\pi R_x$ and $\omega_y = 2\pi R_y$ (as usual, R_x and R_y are the spatial frequencies). This Fourier transform can be written in the form,

$$b(\omega_x, \omega_y) = \frac{1}{2\pi} \int \int_{-\infty}^{\infty} B(X, Y) \exp[i(\omega_x X + \omega_y Y)] \, dx dy$$
$$= F\{B(X, Y)\}. \tag{3}$$

The Fourier frequencies in this are given as follows:

$$\omega_x = 2\pi R_x = -\frac{2\pi}{\lambda} \frac{p}{f'_1} = \frac{2\pi}{\lambda} \sin \sigma_x, \tag{4a}$$

$$\omega_y = 2\pi R_y = -\frac{2\pi}{\lambda} \frac{q}{f'_1} = \frac{2\pi}{\lambda} \sin \sigma_y, \tag{4b}$$

The inverse Fourier transform $F^{-1}\{b(\omega_x, \omega_y)\}$ then represents the original object function

$$B(X, Y) = F^{-1}\{b(\omega_x, \omega_y)\} = \frac{1}{2\pi} \int \int_{-\infty}^{+\infty} b(\omega_x, \omega_y) \exp[-i(\omega_x X + \omega_y Y)] \, d\omega_x d\omega_y. \tag{5}$$

To ensure that the image of the object produced by the microscope and termed B' below is in all points identical with the object, the Fourier spectrum of the object $b(\omega_x, \omega_y)$ should remain unaffected by the instrument.

Even with the most perfect microscope this cannot be achieved. The least which is likely to occur is a cut-off of the Fourier spectrum $b(\omega_x, \omega_y)$ by the aperture stop. A change may also be brought about by the wave aberrations of the microscope and, for example, in phase-contrast microscopy by the phase plate annulus in the pupil. Calculation of the image produced by the microscope must be based on a Fourier spectrum changed by the instrument, which we term $b'(\omega_x, \omega_y)$. This changed Fourier spectrum can be represented by the product of the undisturbed Fourier spectrum with a function $g(\omega_x, \omega_y)$,

$$b'(\omega_x, \omega_y) = b(\omega_x, \omega_y) g(\omega_x, \omega_y) \tag{6}$$

g is an important function of the microscope.

In the simplest case g is a Π-function or rectangular function if 'manipulations in the pupil' consist merely of the existence of an aperture stop so that the aperture angle σ has an upper limit but the instrument is free from opto-geometrical image aberrations.

The Π-function in the Fourier plane (that is the back focal plane of the objective) is shown in figure 3 for the case of a rectangular aperture stop with the dimensions $2p_0 \times 2q_0$. The aperture stop defines the upper cut-off frequency ω_g and the aperture angle σ_0.

In terms of network theory the microscope is a low pass filter with cut-off frequency ω_g or R_g. The cut-off frequency in the object plane is thus given, because

$$R_g = \frac{\sin \sigma_0}{\lambda}, \tag{7}$$

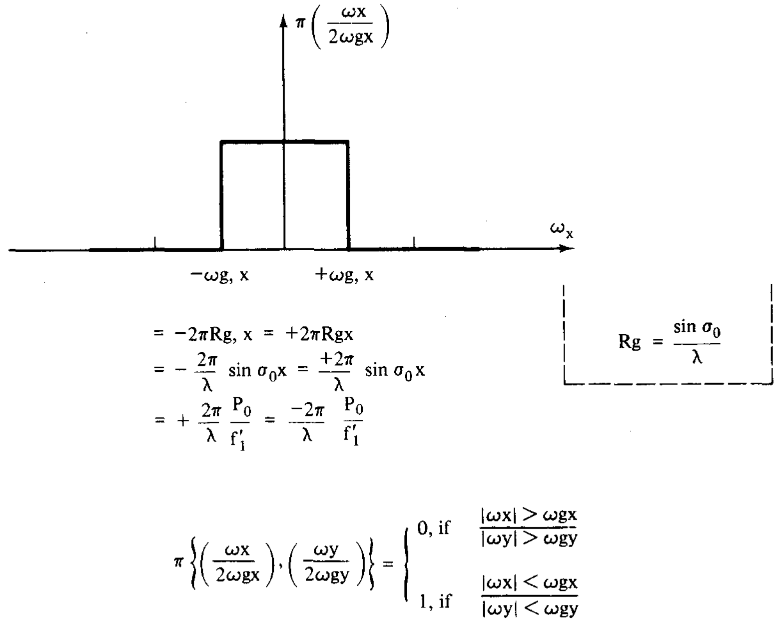

Figure 3. The Π-function in the nomenclature of equations (1) to (4) and figure 2.

or, referred to the vacuum wavelength,

$$R_g = \frac{n \sin \sigma_0}{\lambda}. \tag{8}$$

Its reciprocal is already the resolution limit of periodic objects in coherent light as laid down by Abbe.

The image produced by the microscope is presented by the inverse Fourier transform of $b'(\omega_x, \omega_y)$†,

$$B'(\bar{X}, \bar{Y}) = F^{-1}\{b'(\omega_x, \omega_y)\}$$
$$= \frac{1}{2\pi} \int\int_{-\infty}^{+\infty} b(\omega_x, \omega_y) g(\omega_x, \omega_y) \exp\left[-i(\omega_x X + \omega_y Y)\right] d\omega_x d\omega_y. \tag{9}$$

When the inverse Fourier transform of $g(\omega_x, \omega_y)$ is represented by $G(X, Y)$, it follows that

$$G(X, Y) = F^{-1}\{g(\omega_x, \omega_y)\}$$
$$= \frac{1}{2\pi} \int\int_{-\infty}^{+\infty} g(\omega_x, \omega_y) \exp\left[-i(\omega_x X + \omega_y Y)\right] d\omega_x d\omega_y. \tag{10}$$

† On the scale of the coordinates X, Y of the object plane. It applies to the corresponding coordinates of the image plane X', Y' where $X' = \beta \bar{X}$, $Y' = \beta \bar{Y}$ (β = lateral magnification, in figure 2: $\beta = -f_2/f_1$). Abbe termed the light distribution in the Fourier plane $b(p, q)$ or $b(\omega_x, \omega_y)$ "the primary image" and the microscopic image $B'(X', Y')$ in the image plane "the secondary image".

The result will be the microscopic image $B'(\bar{X}, \bar{Y})$ as a convolution integral according to the convolution theorem of the Fourier transform,

$$B'(\bar{X}, \bar{Y}) = \frac{1}{2\pi} \int\int_{-\infty}^{+\infty} B(X, Y) G[(\bar{X}-X), (\bar{Y}-Y)]\, dX dY. \tag{11}$$

In words, the microscopic image is the result of a convolution of the object function with the Fourier transform of the instrument function. The instrument function is identical with the optical transfer function.

For ideal microscopic imaging the instrument function $g(\omega_x, \omega_y)$, that is the optical transfer function, is a rectangular function Π, and its (inverse) Fourier transform is the function $\sin x/x$ or $\operatorname{sinc}(x)$. Using the terminology of this paper, $G(X, Y)$ has the form

$$G(X, Y) = \frac{2}{\pi} \omega_{gx} \omega_{gy} \operatorname{sinc}(\omega_g X) \operatorname{sinc}(\omega_g Y), \tag{12 a}$$

or

$$G(X, Y) = C \operatorname{sinc}\left(\frac{2\pi}{\lambda} X \sin \sigma_{0x}\right) \operatorname{sinc}\left(\frac{2\pi}{\lambda} Y \sin \sigma_{0y}\right). \tag{12 b}$$

The function $G(X)$ according to equations (12 *a*) and (12 *b*) is identical with the light distribution function of the image of a point object with rectangular aperture, because equations (10) and (12) also describe the Fraunhofer diffraction phenomenon with rectangular aperture for light emitted by a point object. We therefore call $G(X, Y)$ the 'point-image function'.

We note that in a microscope the image is produced by the convolution of the object function with the point-image function. Consequently, when the formalism of the Fourier transform is applied, Abbe's theory is presented in a consistent form.

Of the objects whose microscopic images Abbe calculated (see p. 1692), only one example is treated below according to equations (11) and (12).

Two narrow slits at a finite distance

(*a*) *Coherent light*

We follow Abbe in treating the problem one dimensionally in the X-direction. Let the slits be represented as two δ-functions separated by a distance Δ, symmetrical about the coordinate origin. If α is the angle of incidence of the illumination and is set such that

$$k \sin \alpha = \omega_0, \tag{13}$$

it follows for the object function that

$$B(X) = \delta(X - X_1) \exp(-i\omega_0 X_1) + \delta(X + X_1) \exp(i\omega_0 X_1). \tag{14}†$$

With the point-image function $G(X)$ given by equation (12 *a*), it follows, for the image function of the double slit after the convolution operation, that

$$B'(\bar{X}) = \{\operatorname{sinc}[\omega_g(\bar{X} - X_1)] + \operatorname{sinc}[\omega_g(\bar{X} + X_1)] \exp(2i\omega_0 X_1)\} \exp(-i\omega_0 X_1). \tag{15}$$

† The right-hand side of equation (14) should be multiplied by a factor having the same dimensions of 'light amplitude' (complex amplitude of one vector of the field). For reasons of simplicity this factor is set equal to 1.

The resulting intensity of the microscopic image of the two slits is obtained by taking the squared modulus of equation (15). With $X_1 = \Delta/2$ this yields

$$I_{\rm res} = \left\{ \operatorname{sinc}\left[\omega_g\left(\bar{X} - \frac{\Delta}{2} \right) \right] \right\}^2 + \left\{ \operatorname{sinc}\left[\omega_g\left(\bar{X} + \frac{\Delta}{2} \right) \right] \right\}^2$$
$$+ 2 \operatorname{sinc}\left[\omega_g(\bar{X} - \frac{\Delta}{2}) \right] \operatorname{sinc}\left[\omega_g\left(\bar{X} + \frac{\Delta}{2} \right) \right] \cos\left(\frac{2\pi}{\lambda} \Delta \sin\alpha \right). \quad (16)$$

(*b*) *Incoherent light*

In this case we have a point-image function

$$\tilde{G} = |\operatorname{sinc}(\omega_g X)|^2. \quad (17)$$

Since in the case of incoherent illumination all light fluctuations are completely uncorrelated, interference between the two slits is impossible. The total intensity is therefore the sum of the intensities of the two slits, which are, of course, independent of the angle of illumination. Thus the object function can be applied to incoherent illumination in the following way:

$$B(X) = \delta(X - X_1) + \delta(X + X_1) \quad (18)$$

The right-hand side of equation (18) should be multiplied by a factor having the dimensions of light intensity. This factor is set equal to 1. In contrast to equation (14) $B(X)$ in equation (18) is a function of the light intensity. The convolution integral will then have the form

$$B'(\bar{X}) = I_{\rm res} = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} \{\delta(X - X_1) + \delta(X + X_1)\}\{\operatorname{sinc}[\omega_g(\bar{X} - X)]\} dX. \quad (19)$$

After evaluation we obtain

$$B'(\bar{X}) = I_{\rm res} = \left\{ \operatorname{sinc}\left[\omega_g(\bar{X} - \frac{\Delta}{2}) \right] \right\}^2 + \left\{ \operatorname{sinc}\left[\omega_g(\bar{X} + \frac{\Delta}{2}) \right] \right\}^2. \quad (20)$$

Abbe treated equations (16) and (20) for the real case with the then available methods, and arrived, of course, at the same results. Equations (16) and (20) can therefore be interpreted according to the original images in the book by Lummer and Reiche: figure 4 shows the results for coherent illumination. It is obvious that in incoherent light the double-slit structure is resolved with a slit separation of $\Delta = \lambda/(2 \sin \sigma)$. In coherent light the double-slit structure with the same separation is still not resolved. If in oblique coherent light the cosine in the third term assumes the value 0 or -1, the double-slit structure is also resolved at a separation of $\Delta = \lambda/(2 \sin \sigma)$.

A detailed discussion of the microscopic imaging of a double slit can be found in the relevant literature (see, for example, [1, 13]). We note that coherent illumination parallel to the axis results in less resolving power than incoherent illumination and that the *r* resolving power with incoherent illumination is only achieved for certain angles of incidence with coherent light†. For the treatment of other microscopic objects such as a slit of finite width or periodic objects, see in particular [1].

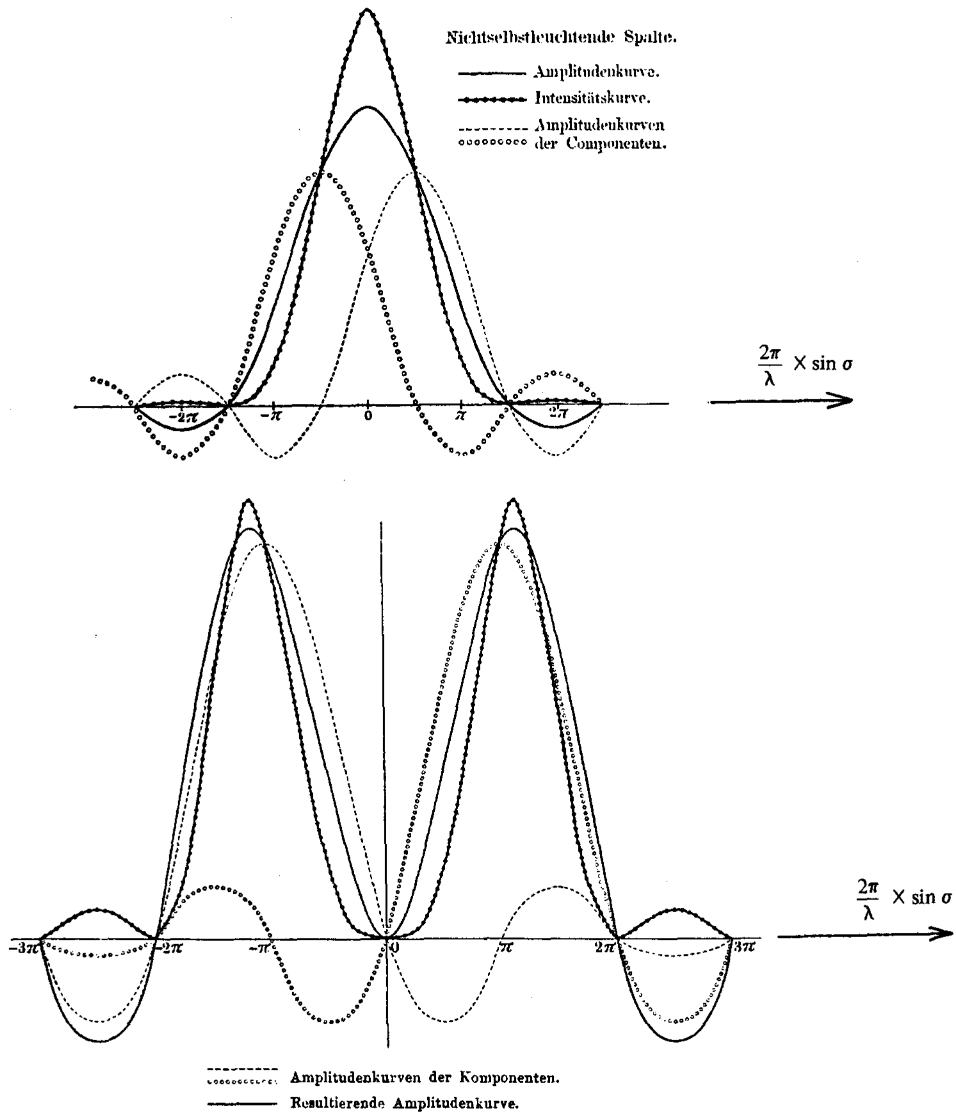

Figure 4. Microscopic image of two narrow slits with a separation Δ in coherent light parallel to the axis according to Abbe (reproduced from [8]). Top: slit separation $\Delta = \lambda/(2\sin\sigma)$; and bottom: slit separation $\Delta = \lambda/\sin\sigma$.

† With respect to the intensity curve for the image of a double slit, presented in figure 5, with a 19 per cent dip in the centre $\lambda/(2\sin\sigma)$ is since Abbe's time generally referred to as the 'limit of resolution' of the microscope. This is, however, a conventional but not a physical definition. Whether or not two slits with a separation smaller or larger than $\lambda/(2\sin\sigma)$ are resolved is dependent on the sensitivity of the detector. It is possible today with suitable observation equipment to 'resolve' double slit structures with much smaller slit separations. The decisive finding is that the 'resolving power' is proportional to λ and inversely proportional to $\sin\sigma$.

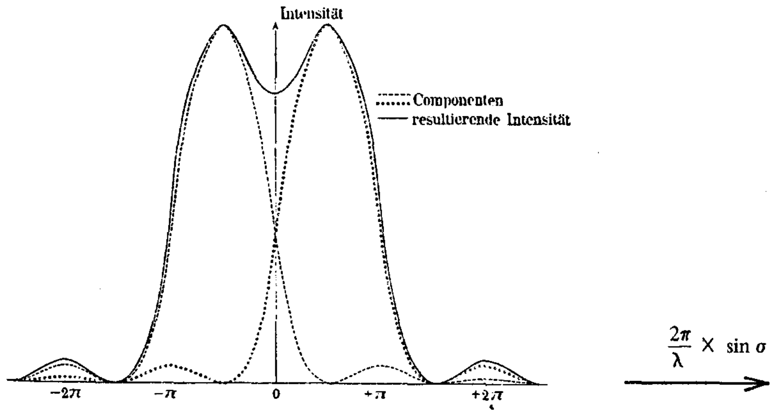

Figure 5. Microscopic image of two narrow slits with a separation $\Delta = \lambda/(2\sin\sigma)$ in incoherent light according to Abbe (reproduced from [8]).

4. Conclusion

The most important result of Abbe's work on the theory of microscopic image formation was undoubtedly the fact that he was the first to realize that the resolving power of the microscope is also limited by the diffraction phenomenon and thus by the objective aperture, a fact which had been known for decades as far as telescopes were concerned.

Abbe's theory of image formation in the microscope is primarily a theory of microscopic imaging in coherent light and in addition a theory of coherent imaging in general. Microscopic imaging in incoherent light is treated along with the microscopic imaging of double slits.

In Abbe's time the essential difference between image formation in coherent and incoherent light was a completely new piece of knowledge, which represents another important result of Abbe's efforts.

It was only on the foundation Abbe had laid that Zernike could develop the principles of phase-contrast microscopy and Gabor, Leith and Upatnieks those of holography and the modern techniques of coherent image processing, pattern recognition, etc.

Abbe's contemporaries were unable to realize how much they were indebted to him for his discovery. Until the 1920s depreciative comments were published over and over again, which discounted the study of microscopic imaging in coherent light as unimportant and impractical [14–19].

Yet, the development proved the superior quality and importance of Abbe's theory which goes far beyond microscopic imaging proper. From our point of view the unfair criticism was altogether unfounded.

Une étude bibliographique historique des publications, entre 1873 et 1910, concernant la théorie d'Abbe de la formation des images dans le microscope est présentée. On en donne, ensuite, la théorie d'après l'algorithme de la transformée de Fourier complexe.

Diese Arbeit gibt einen historischen Überblick über Veröffentlichungen zur Abbeschen Theorie aus den Jahren 1873 bis 1910. Dabei wird Abbes Theorie mit Hilfe der komplexen Fouriertransformation präsentiert.

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